A Three-Phase UPFC Model for Power Flow Control in Unbalanced Transmission Networks

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***Abstract*--This paper presents a phase co-ordinate steady-state model of the Unified Power Flow Controller (UPFC) suitable for power flow control in unbalanced transmission networks. The UPFC power flow equations are integrated into an existing Newton-Raphson power flow program to assess its active and reactive power control performance. The electric system and UPFC equations are solved in a unified single-frame of reference such that nodal voltage state variables and UPFC control settings are computed simultaneously through the iterative process. Hence, the algorithm exhibits quadratic convergence characteristic. Guidelines and methods for implementing the UPFC and its adjustments within the Newton-Raphson algorithm are also described. A set of analytical equations has been derived to provide good UPFC initial conditions. Validation of the proposed model is carried out in a benchmark network operating under balanced conditions. Test results are presented which demonstrate the prowess of the proposed model to control active and reactive power flow in both balanced and unbalanced power network operating conditions.**

***Index Terms*- FACTS, Power Flow Control, Newton Raphson, Unified Power Flow Controller.**

1. Introduction

Electric utility industry is undergoing dramatic changes due to privatisation, deregulation and competition in electricity markets in many leading countries. As a consequence, increasing wheeling transactions lead to a growing demand of transfer capabilities. In this new environment, a demand for flexible power flow control is becoming a technical need because the increase in the level of risk and uncertainty associated with transmission operation and investment. One response to the flexibility in the way that the transmission system is operated is found by applying the Flexible AC Transmission Systems (FACTS) concept [1]. FACTS is a title used to encompass all the newly emerging high-voltage controllers based on leading-edge power electronics technology to enhance the controllability and increase power transfer capacity of a transmission system. Among different proposals of FACTS controllers, the UPFC is regarded as the most versatile and generalised controller envisaged in recent years [2,4].

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In order to assess the impact of the UPFC on the steady-state electric transmission system operation, it is necessary to develop its mathematical model and include it in a power flow program. While a significant amount of work has been and continues to be devoted on the modelling and analysis of the UPFC performance, it has been restricted to positive sequence domain [4-8] and none attention has been paid to carry out modelling and analysis in the phase co-ordinates domain.

The goal of this paper is a derivation of a mathematical steady-state model of the UPFC in the natural framework of electric systems, i.e. phase co-ordinate domain. The proposed model is suitable for active and reactive power flow control in transmission networks operating under either balanced or unbalanced conditions. An existing NR based polyphase power flow program has been upgraded to include the proposed model. The power flow equations pertaining to the UPFC and the power network are solved in a unified single-frame of reference leading a very robust iterative solution. In this unified solution, nodal voltage state variables and UPFC control settings are computed simultaneously through the iterative process. Hence, the algorithm exhibits quadratic convergence characteristic regardless of the network size and UPFC controllers embedded in the network. Guidelines and methods for implementing the UPFC and its adjustments within the Newton-Raphson algorithm are also described. A set of analytical equations has been derived to provide good UPFC initial conditions. Validation of the proposed model is carried out in a benchmark network operating under balanced conditions. Test results are presented which demonstrate the prowess of the proposed model to control active and reactive power flow in both balanced and unbalanced power network operating conditions.

2. Basic principles of UPFC

The basic principles of UPFC operation are already well established in the open literature [2-4]. Fig. 1 shows a single-line system configuration of a general UPFC. The UPFC is composed of two back-to-back self-commutated, voltage source converters (VSC) operated from a common dc link capacitor. The converters are coupled to the network via a shunt (exciting) and series (boosting) transformer. The series VSC is used to generate a fundamental frequency voltage with variable magnitude and phase angle which is added to the AC transmission line by the series connected boosting transformer, as shown in Fig. 2. In this way, depending upon magnitude ($V_{cRmin} \le V_{cR} \le V_{cRmax}$) and/or phase-angle ($0 \le q_{cR}$

≤2π) values of $\boldsymbol{V_{cR}}$, the active and reactive power flow injected at the receiving-end of the transmission line can be regulated independently each other. Hence, the series VSC acts as a combination of voltage regulator, variable series compensator and phase shifter. This VSC can internally generate or absorb reactive power at its AC terminal and only the active power has to be supplied at its DC input terminal from the AC power system. This power exchange results in either the discharge or overcharge of the DC link capacitor. To maintain the capacitor voltage at the required value, and assure proper operation of the series VSC, the shunt converter is used to regulate the amount of active power drawn from the AC system at the common DC link terminal. In addition, this shunt VSC has the capability of controlling the reactive power at its AC terminal, independently the active power it transfers to (or from) the DC terminal.

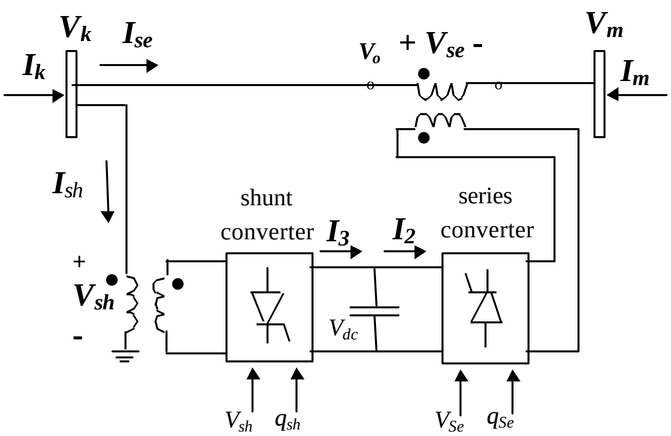

Fig. 1. Single line UPFC schematic diagram.

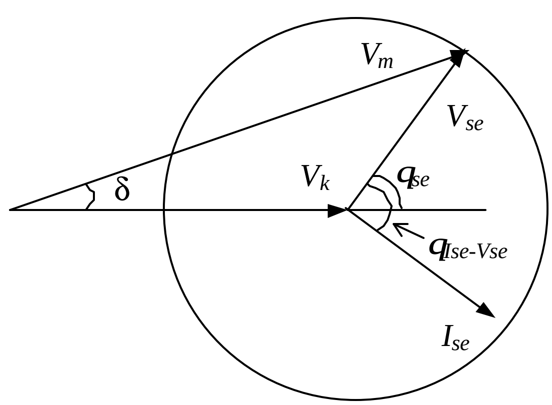

Fig. 2. Phasor diagram of the UPFC system of Fig. 1.

3. Phase domain power flow model

The primary and main function of the UPFC is an independent control of the active and reactive power flow injected into the transmission line. Bearing this in mind, the shunt VSC can be assumed to operate at unity power factor such that its only function is to provide a closed path to supply the active power demanded by the series SVC at its DC terminals. Under this situation, the shunt converter can be disregarded from the UPFC circuit analysis and the UPFC controller is well represented by an ideal series voltage source [2,3,8,9], termed Series Synchronous Voltage Source (SSVS).

Figure 3 shows a schematic diagram of a three phase transmission line compensated by a UPFC. The complex voltage injected by the SSVS has variable magnitude and phase-angle which adjust automatically so as to control the active and reactive powers exchanged by the UPFC and the AC system.

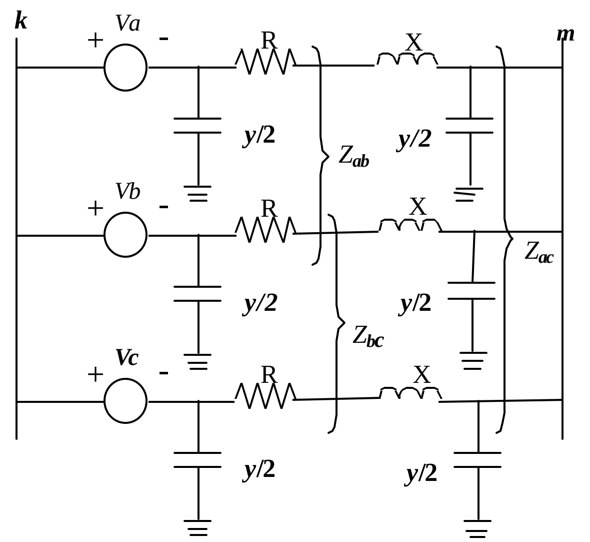

Fig. 3. Transmission line compensated by a UPFC.

The ideal SSVS representing the UPFC is :

$$\mathbf{V}_{se}^{\rho} = V_{se}^{\rho}\,(\cos\,\theta_{se}^{\rho} + j\ \mathrm{sen}\ \theta_{se}^{\rho}\,) \tag{1}$$

The variable ρ takes the values *a,b* and *c* corresponding to the three phases involved, i.e. ρ = *a,b,c*. Furthermore, V_{se}^{ρ} and θ_{se}^{ρ} are the controllable magnitude ($V_{se\ min}^{\rho} \le V_{se}^{\rho} \le V_{se\ max}^{\rho}$) and phase-angle ($0 \le \theta_{se}^{\rho} \le 2\pi$), respectively.

The general transfer admittance matrix for this configuration is,

$$\begin{bmatrix} \boldsymbol{I}_k^{abc} \\ \boldsymbol{I}_m^{abc} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_{kk}^{abc} & -\boldsymbol{Y}_{kk}^{abc} & \boldsymbol{Y}_{km}^{abc} \\ \boldsymbol{Y}_{km}^{abc} & -\boldsymbol{Y}_{kk}^{abc} & \boldsymbol{Y}_{mm}^{abc} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_k^{abc} \\ \boldsymbol{V}_{se}^{abc} \\ \boldsymbol{V}_m^{abc} \end{bmatrix} \tag{2}$$

where $\boldsymbol{Y}_{ij}^{\rho}$ is the transmission line admittance matrix with real and imaginary elements given by

$$\left[\boldsymbol{Y}_{ij}^{abc}\right] = \begin{bmatrix} G_{ij}^{aa} & G_{ij}^{ab} & G_{ij}^{ac} \\ G_{ij}^{ba} & G_{ij}^{bb} & G_{ij}^{bc} \\ G_{ij}^{ca} & G_{ij}^{cb} & G_{ij}^{cc} \end{bmatrix} + j \begin{bmatrix} B_{ij}^{aa} & B_{ij}^{ab} & B_{ij}^{ac} \\ B_{ij}^{ba} & B_{ij}^{bb} & B_{ij}^{bc} \\ B_{ij}^{ca} & B_{ij}^{cb} & B_{ij}^{cc} \end{bmatrix} \tag{3}$$

where *i* and *j* take the values *k* and *m*, corresponding to the nodes involved.

Based on the equivalent circuit shown in Fig. 3, the active and reactive power equations of the integrated three phase transmission line and UPFC at node *k* are:

$$P_k^p = V_k^p \left\{ \sum_{i=k,m} \sum_{j=a,b,c} V_i^p \left[G_{ki}^{pj} \cos\left(\theta_k^p - \theta_i^j\right) + B_{ki}^{pj} sen\left(\theta_k^p - \theta_i^j\right) \right] - V_s^j \left[G_{kk}^{pj} \cos\left(\theta_k^p - \theta_s^j\right) + B_{kk}^{pj} sen\left(\theta_k^p - \theta_s^j\right) \right] \right\} \quad (4)$$

$$Q_k^p = V_k^p \left\{ \sum_{i=k,m} \sum_{j=a,b,c} V_i^j \left[G_{ki}^{pj} sen\left(\theta_k^p - \theta_i^j\right) + B_{ki}^{pj} \cos\left(\theta_k^p - \theta_i^j\right) \right] - V_s^j \left[G_{kk}^{pj} sen\left(\theta_k^p - \theta_s^j\right) + B_{kk}^{pj} \cos\left(\theta_k^p - \theta_s^j\right) \right] \right\} \quad (5)$$

A similar set of equations are obtained to node *m* by exchanging *k* and *m*.

4. Solution of UPFC/AC system power flow equations

The solution of the conventional polyphase power flow equations is already well documented in the open literature [10]. However, the solution of these equations with FACTS controllers embedded in the network is not a trivial matter. In this context, the problem is formulated as solving a set of non-linear algebraic equations of the form

$$F(x_{ac}, x_{upfc}) = 0 \quad (6)$$

$$G(x_{ac}, x_{upfc}) = 0 \quad (7)$$

where (6) and (7) describe the AC system and UPFC power flow equations, respectively. There are basically two main different approaches to solve this problem which have been applied for positive sequence studies, the sequential method and the unified method [4]. In both approaches, these equations are generally solved by the Newton-Raphson (NR) method. The sequential approach decouples the controller from the power system network by transferring the equivalent voltage sources as node power injections. Hence, the non-linear effect of the UPFC is included only in the power mismatch equations [5] or both power mismatch equations and a few elements of the Jacobian matrix [4,6,7]. After the power flow computation is carried out, the UPFC control parameters are determined by solving a mini-scale non-linear equations [4,5]. Despite that the additional computation burden incurred in this approach is minimal, a major drawback is that it is not possible to assess during the power flow iteration process if the UPFC parameters go beyond limits. Then, after the power flow convergence, the computed parameters of the UPFC may result in a violated solution.

On the other hand, the unified approach adds the UPFC controllable parameters with the magnitudes and angles of all nodal voltages into a solution vector $\boldsymbol{x}$ [8]. The UPFC and AC system power flow mismatch equations, grouped together in the vector $\boldsymbol{f(x)}$, are linearised by the NR method around a base point and are solved simultaneously. The set of algebraic linear equations $\boldsymbol{J\Delta x = -f(x)}$ is solved iteratively for the state variable deviation vector $\boldsymbol{\Delta x}$ starting from an initial guess $\boldsymbol{x^0}$. In this unified solution, the UPFC control parameters are adjusted simultaneously with the AC system state variables in order to achieve the specified control targets. Although the vectors $\boldsymbol{f(x)}$, $\boldsymbol{\Delta x}$, as well as Jacobian matrix $\boldsymbol{J}$ must to be enlarge to include the UPFC effect, from a convergence point of view, this approach is superior to the sequential method because it has control over UPFC limits at each iteration [9]. Thus the unified approach presented in [8,9] is extended in this paper to implement the UPFC controller in a three phase power flow algorithm .

According to the aforementioned, the linearisation of equations (6) and (7) by the NR method results in

$$\begin{bmatrix} F(x_{ac}, x_{upfc}) \\ G(x_{ac}, x_{upfc}) \end{bmatrix} = \begin{bmatrix} \boldsymbol{J_1} & \boldsymbol{J_2} \\ \boldsymbol{J_3} & \boldsymbol{J_4} \end{bmatrix} \begin{bmatrix} \Delta x_{ac} \\ \Delta x_{upfc} \end{bmatrix} \quad (8)$$

where $\boldsymbol{J_1}$ is the conventional Jacobian matrix and,

$$\boldsymbol{J_2} = \frac{\partial F(x_{ac}, x_{upfc})}{\partial x_{upfc}} \quad (9)$$

$$\boldsymbol{J_3} = \frac{\partial G(x_{ac}, x_{upfc})}{\partial x_{ac}} \quad (10)$$

$$\boldsymbol{J_4} = \frac{\partial G(x_{ac}, x_{upfc})}{\partial x_{upfc}} \quad (11)$$

Equation (8) can be expended based on Fig. 3. The set of linearised equations to be solved is,

$$\begin{bmatrix} \Delta P_k^r \\ \Delta P_m^r \\ \Delta Q_k^r \\ \Delta Q_m^r \\ \Delta P_{km}^r \\ \Delta Q_{km}^r \end{bmatrix}^i = \begin{bmatrix}
\frac{\partial P_k^r}{\partial \theta_k^r} & \frac{\partial P_k^r}{\partial \theta_m^r} & \frac{\partial P_k^r}{\partial V_k^r} V_k^r & \frac{\partial P_k^r}{\partial V_m^r} V_m^r & \frac{\partial P_k^r}{\partial \theta_s^r} & \frac{\partial P_k^r}{\partial V_s^r} V_s^r \\
\frac{\partial P_m^r}{\partial \theta_k^r} & \frac{\partial P_m^r}{\partial \theta_m^r} & \frac{\partial P_m^r}{\partial V_k^r} V_k^r & \frac{\partial P_m^r}{\partial V_m^r} V_m^r & \frac{\partial P_m^r}{\partial \theta_s^r} & \frac{\partial P_m^r}{\partial V_s^r} V_s^r \\
\frac{\partial Q_k^r}{\partial \theta_k^r} & \frac{\partial Q_k^r}{\partial \theta_m^r} & \frac{\partial Q_k^r}{\partial V_k^r} V_k^r & \frac{\partial Q_k^r}{\partial V_m^r} V_m^r & \frac{\partial Q_k^r}{\partial \theta_s^r} & \frac{\partial Q_k^r}{\partial V_s^r} V_s^r \\
\frac{\partial Q_m^r}{\partial \theta_k^r} & \frac{\partial Q_m^r}{\partial \theta_m^r} & \frac{\partial Q_m^r}{\partial V_k^r} V_k^r & \frac{\partial Q_m^r}{\partial V_m^r} V_m^r & \frac{\partial Q_k^r}{\partial \theta_s^r} & \frac{\partial Q_k^r}{\partial V_s^r} V_s^r \\
\frac{\partial P_{km}^r}{\partial \theta_k^r} & \frac{\partial P_{km}^r}{\partial \theta_m^r} & \frac{\partial P_{km}^r}{\partial V_k^r} V_k^r & \frac{\partial P_{km}^r}{\partial V_m^r} V_m^r & \frac{\partial P_{km}^r}{\partial \theta_s^r} & \frac{\partial P_{km}^r}{\partial V_s^r} V_s^r \\
\frac{\partial Q_{km}^r}{\partial \theta_k^r} & \frac{\partial Q_{km}^r}{\partial \theta_m^r} & \frac{\partial Q_{km}^r}{\partial V_k^r} V_k^r & \frac{\partial Q_{km}^r}{\partial V_m^r} V_m^r & \frac{\partial Q_{km}^r}{\partial \theta_s^r} & \frac{\partial Q_{km}^r}{\partial V_s^r} V_s^r
\end{bmatrix}^i \begin{bmatrix} \Delta \theta_k^r \\ \Delta \theta_m^r \\ \frac{\Delta V_k^r}{V_k^r} \\ \frac{\Delta V_m^r}{V_m^r} \\ \Delta \theta_s^r \\ \frac{\Delta V_s^r}{V_s^r} \end{bmatrix}^i \quad (12)$$

where all elements of equation (12) are matrices of third order and *i* is the iteration number.

The UPFC state variables are updated at the end of each iterative step according to the following equations,

$$V_s^{r(i+1)} = V_s^{r(i)} + \left(\frac{\Delta V_s^r}{V_s^r} \right)^{(i)} V_s^{r(i)} \quad (13)$$

$$\theta_s^{r(i+1)} = \theta_s^{r(i)} + \Delta \theta_s^{r(i)} \quad (14)$$

4 Practical implementation of the UPFC model

It is clear from (4) and (5) that power flow equations are quadratic in terms of voltages and involve sinusoidal functions of voltage angles. Therefore, there are many solutions, that is, many equilibrium points where the power mismatch equations are satisfied. The choice of the initial estimation of the UPFC control parameters and the numerical properties of the Jacobian matrix evaluated at each iteration have an important bearing upon the iterative process and convergence towards a practical solution.

4.1 Initial Conditions

It is widely recognised that conventional power flow algorithms are convergence fail-prone when applied to power systems with embedded facts devices [4,8]. In order to avoid a degradation of the Newton quadratic convergence characteristic, it is necessary to provide good starting conditions for the UPFC state variables. They can be estimated from the UPFC power equations (4) and (5) assuming a lossless transmission line and a flat voltage profile at the compensated transmission line nodes. According to the active power P_{ref}^{r} and reactive power Q_{ref}^{r} to be controlled, the following set of analytical equations are obtained neglecting the coupling between phases [11].

When the active and reactive powers are regulated from node k to node m, the mathematical expressions for the UPFC initial conditions are,

$$V_s^{r} = \frac{\pm\sqrt{{P_{kref}^{r}}^{2} + {C1^{r}}^{2}}}{{V_k^{r}}^{0} B_{kk}^{rr}} \qquad (7)$$

$$\theta_s^{r} = Tan^{-1}\left(\frac{P_{kref}^{r}}{C1^{r}}\right) \qquad (8)$$

where the superscript 0 indicates initial value and

$$C1^{r} = Q_{kref}^{r} + \left(V_k^{r\,0}\right)^2 B_{kk}^{rr} + V_k^{r\,0}\, V_m^{r\,0}\, B_{km}^{rr} \qquad (8)$$

When the active and reactive powers are regulated from node m to node k, the mathematical expressions for the UPFC initial conditions are obtained by exchanging k and m in (6) to (8).

When the active and reactive power are not define at the same node, the additional assumption of neglecting the transmission line shunt capacitance should be considers in order to obtain analytical equations to the initial conditions.

The following equations are used to initialised the UPFC if the active and reactive powers are specified at nodes k and m, respectively.

$$V_s^{r} = \frac{\pm\sqrt{\left(\frac{P_{kref}^{r}}{V_m^{r0}}\right)^2 + \left(\frac{C1^{r}}{V_k^{r0}}\right)^2}}{B_{km}^{rr}} \qquad (9)$$

$$\theta_s^{r} = -Tan^{-1}\left(\frac{V_m^{r\,0} P_{kref}^{r}}{V_k^{r\,0} C1^{r}}\right) \qquad (10)$$

where

$$C1^{r} = Q_{mref}^{r} + V_m^{r\,0}\, B_{km}^{rr}\left(V_k^{r\,0} - V_m^{r\,0}\right) \qquad (11)$$

If the active and reactive powers are specified at nodes m and k, respectively, the initial conditions equations are obtained from (9) to (11) by exchanging k and m.

Following the same line of reasoning, it is possible to obtain the UPFC initial conditions but considering the mutual coupling between the transmission line phases. By way of example, when the active and reactive powers are regulated from node k to node m, the mathematical expressions for the UPFC initial conditions are [11],

$$V_s^{r} = \frac{\sqrt{\left(C1^{r}\right)^2 + \left(C2^{r}\right)^2}}{V_k^{r} \sum_{i=a,b,c} B_{kk}^{ri}} \qquad (12)$$

$$\theta_s^{p} = -Tan^{-1}\left(\frac{C1^{r}}{C2^{r}}\right) + \theta_k^{p} \qquad (13)$$

$$C1^{r} = P_k^{r} - V_k^{r} \sum_{j=k,m} \sum_{\substack{i,l=a,b,c \\ i\neq l\neq r}} \left(V_j^{i} B_{kj}^{fi} - V_j^{l} B_{kj}^{fl}\right) \qquad (14)$$

$$C2^{r} = Q_k^{r} + V_k^{r} \sum_{j=k,m} \sum_{i=a,b,c} V_j^{i} B_{kj}^{fi} \qquad (15)$$

Owing to the fact that the proposed equations compute the UPFC initial parameters based on the power control targets and nodal voltages, they increase the chance of starting the iterative process in the vicinity of the wanted solution.

4.2 Truncated adjustments

As pointed out in [9], large increments on the state variable deviation terms Δx during the backward substitution process may induce large increments in the power mismatch terms ***f(x)***. This condition may in turn slow down, or more seriously cause divergence, of the iterative process. The strategy adopted in the program in order to circumvent this problem is limiting the size of the computed adjustments during the backward substitution. In the case that a state variable has a large increment, the computed adjustment is replaced by a truncated adjustment and its effect is propagated throughout the remaining of the backward substitution. Based on extensive experimentation, the maximum step size adjustment chosen for our implementation was $\pm$ 0.1 pu for state variable magnitudes and $\pm 5^{o}$ for state variable angles.

4.3 Starting criterion for limits revision.

The starting criterion defines when the revision process of the control variable limits should commence during the iterative solution. The UPFC control power mismatch equations are used as the guiding principle for conducting its limit revisions. The checking is activated at phase r when either equation (16) or (17) satisfies the pre-defined tolerance of 10^{-3}.

$$\Delta P_{km}^{r} = P_{kref}^{r} - P_{kcal}^{r} \leq TOL \qquad (16)$$

$$\Delta Q_{km}^{r} = Q_{kref}^{r} - Q_{kcal}^{r} \leq TOL \qquad (17)$$

If a voltage magnitude at any phase of the UPFC goes beyond its limit, it is set to its offending limit value. In this case, no further attempts are made to regulate the reactive power along the phase during the rest of the iterative process.

5 Numerical examples

An Object Oriented Programming polyphase load flow program has been extended to incorporate the UPFC model and methods described above. The new program has been applied to the solution of a large number of power networks of different sizes and varying degrees of operational complexity. For purpose of this paper, two networks have been selected to show the prowess of the proposed model.

5.1 Balanced case and model validation

The IEEE 57-bus system [12] has been transformed to a three phase balanced network in order to assess the effectiveness of the proposed model. Two UPCFs were embedded to compensate the transmission lines connected between nodes 4-5 and 44-45, respectively. The controllers were set to increase the active power flow injected at the compensated transmission line phases in 20 %, and to reduce the reactive power flow in equal percentage, as shown in Table 1. The power flow solutions were carried out considering wye-g/wye-g connection in all transformers. The UPFC initial conditions computed based on equations given in Section 4 are shown in Table 2.

Table 1. UPFC power control.

Transmission Line	Original Power (MW)	Original Power (MVARs)	Controlled Power (MW)	Controlled Power (MVARs)
4-5	13.80	-4.43	17.0	-3.54
44-45	-36.52	3.39	-43.8	2.7

Table 2. UPFC initial conditions.

UPFC Controller	UPFC voltage source parameters			
	Magnitude (pu)	Angle (Electrical degrees)		
		Phase *a*	Phase *b*	Phase *c*
1	0.0858	-88.69	-208.69	31.31
2	0.2053	-86.46	-206.46	33.54

To verify the correctness of the proposed model and its implementation, the steady-state solution of this network was also computed using the positive sequence network and UPFC model presented in [8].

The phase co-ordinates and positive sequence power flow solutions were obtained in 6 iterations to a mismatch tolerance of 10^{-12} in both cases. The controllers upheld their target values. Table 3 shows a comparison for the UPFC final parameters. The same results were arrived at. The nodal voltage magnitude and angle profiles are shown in Figures 2 and 3, respectively. As expected, all voltage magnitude solutions lie on top of each other. A similar result takes place with positive sequence and phase "a" nodal voltage angles. Such a coincidence of computation results shows the correctness of the proposed UPFC model and its implementation procedures.

Table 3. UPFC final parameters.

UPFC	UPFC voltage source parameters					
	Magnitude (pu)		Angle (degrees)			
	Sec(+)	Phases a,b,c	Sec(+)	Phases a	Phases b	Phases c
1	0.01132	0.01132	-126.2	-126.2	-246.2	-6.2
2	0.03042	0.03042	57.6	57.6	-62.6	177.6

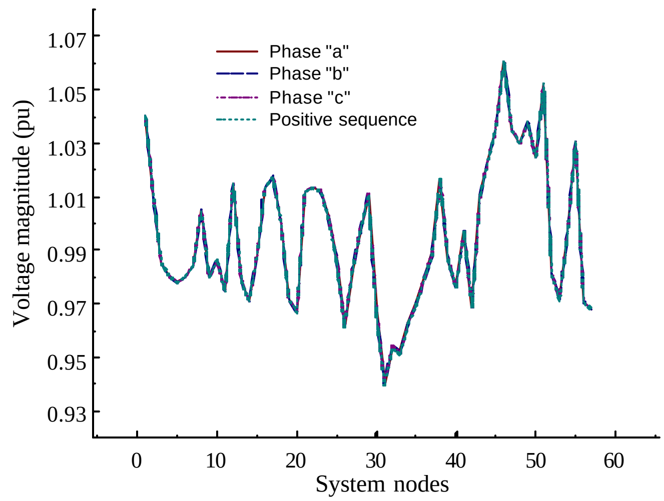

Fig. 2. Comparison of the voltage nodal magnitude.

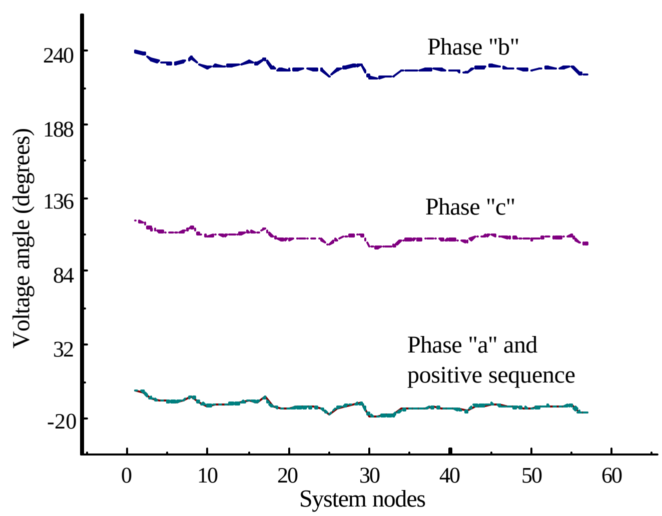

Fig. 3. Comparison of the voltage nodal angle.

5.2 Unbalanced case

To demonstrate the ease with which the proposed UPFC model can be applied to unbalanced systems, the network shown in Fig. 4 has been analysed. The test system corresponds to a combination of a radial and a meshed system [11]. The former is a nine-busbar network corresponding to the lower part of the South Island of New

Zealand [13]. The later is a three phase balanced network corresponding to the Ward-Hale 5-nodes system [14].

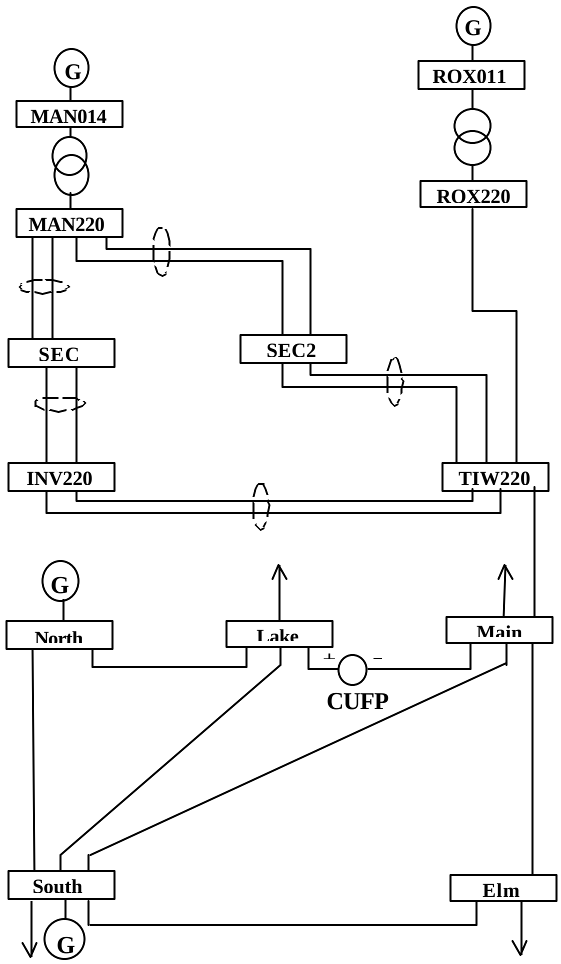

Fig 4. One-line unbalanced 13 nodes system.

The slack node is at man014. The two generator transformers are star-ground/delta with delta winding on the generator side. The generators and transformers are assumed to be balanced. The transmission line models are three phase and include mutual coupling between adjacent circuits, as shown in Fig. 4. Due to the transmission line asymmetry and mutual coupling the system is unbalanced. All phase loads were considered balanced except at node lake were a 20% load unbalance was introduced among phases.

A UPFC has been embedded in the system in order to balance the active and reactive power flowing from Lake to Main. The power flow before the compensation and the UPFC power flow control settings are given in Table 4. The UPFC initial conditions are given in Table 5.

Table 4. Power flow in line Lake-Main.

Line	Phase	Original Flow		Controlled Flow	
		Active (MW)	Reactive (MVar)	Active (MW)	Reactive (MVar)
Lake to Main	A	-41.46	1.650	-50.0	-2.0
	B	-39.69	-1.109	-50.0	-2.0
	C	-57.85	12.437	-50.0	-2.0

Table 5. UPFC initial conditions.

UPFC voltage source parameters			
Magnitude (p.u.) (all phases)	Phase angle (degree)		
	phase A	phase B	phase C
0.0502	-87.74	-207.74	32.36

It must be observed that the UPFC is not only is used for balance the power flow at each phase of the line, but also to divert the reactive power flow along the line phases A and C.

The numerical computation has been performed successfully in 8 iterations to a power mismatch tolerance of 10^{-12} p.u. The UPFC uphold its target value. The UPFC complex voltages are given in Table 6. It is interesting to note that these voltages are unbalanced in order to achieve the specified control.

Table 6. UPFC final conditions .

UPFC voltage source parameters					
Phase magnitude (p.u.)			Phase angle (degrees)		
A	B	C	A	B	C
0.0902	0.0737	0.0967	34.21	14.81	54.94

5.3 Effects of truncated adjustments

The effect of the truncated adjustments on the state variables during the backward substitution is presented in this Section. The same simulation described in the preceding Section was carried out considering full and truncated correction as indicated in Section 4. Figure 5 shows the maximum absolute power mismatches profiles in the UPFC and system's nodes for the two cases under consideration.

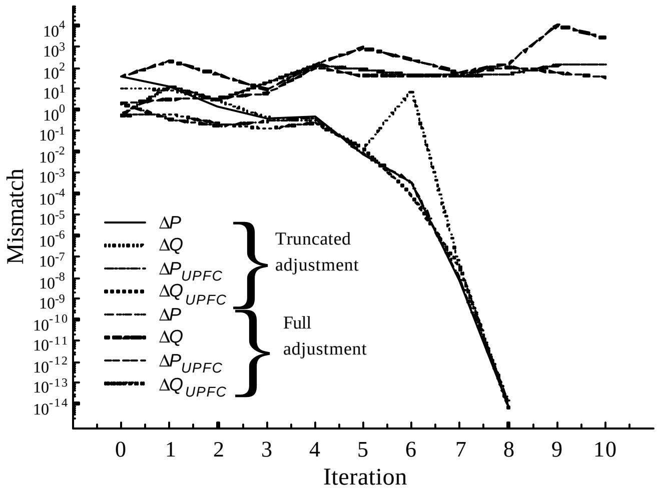

Fig. 5 Convergence profile as function of state variable size increment.

Figures 6 and 7 shows the UPFC voltage magnitude values during the iterative process.

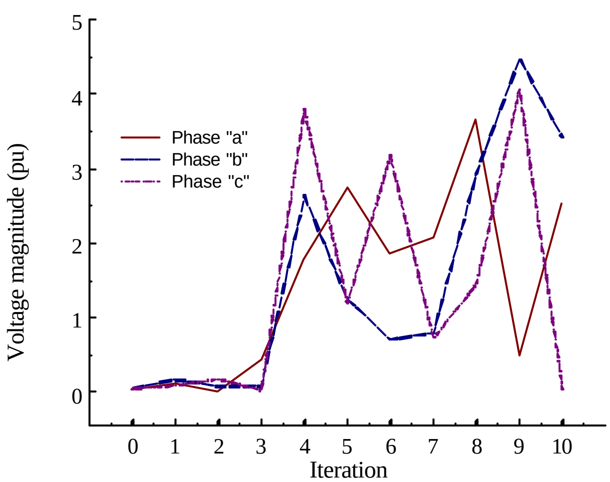

Fig. 6. UPFC voltage magnitude profile as function of iterations (full adjustment).

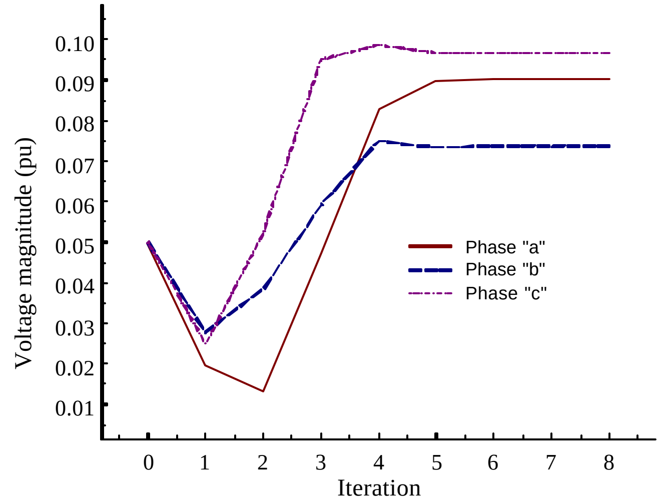

Fig. 7. UPFC voltage magnitude profile as function of iterations (truncated adjustments).

It is observed from Fig. 6 that there is a large overshooting when the size of the state variables is not truncated. This effect produces ill-numerical conditions in the power flow mismatch and Jacobian equations which cause divergence. The very large perturbations which takes place in the UPFC voltage magnitude lead to its limit violations. However, the limits revision did not take place because the starting criterion defined in Section 4.3 was not satisfied. An alternative to carry out the revision is to start adjusting control variable limits at the end of the second iteration [14]. At this point, the UPFC voltage magnitude is revised and suitably enforced. However, this alternative would be inappropriate because the algorithm would converge to an unrealistic solution where the UPFC is not able to achieve its control targets.

On the other hand, the control on the step size of the UPFC and system state variables produce a slow down convergence during the early stages of the solution, as shown in Fig. 7. Nevertheless, this alternative increases the probability of arriving to a realistic solution.

5.4 UPFC initial condition effect

"Ill-Chosen" initial conditions are responsible for the Newton-Raphson load flow solution diverging or arriving to some anomalous solutions. Four different simulations were carried out to assess the impact of UPFC initial conditions on load flow convergence, three cases were solved. The unbalanced network shown in Fig. 4 was used for this purpose. The UPFC control settings are those given in the last section. In all cases was assumed a mismatch tolerance of 10^{-12}. The iterative solutions were started from a flat voltage profile, except at PV nodes. The selected initial conditions and the number of iterations required to converge are summarized in Table 7. The same UPFC final parameters, given in Table 6, were computed in those cases where the solution converged. The convergent trajectories of the maximum absolute nodal power mismatches are shown in Fig. 8 for these cases.

Table 7. UPFC initial conditions.

Cases	UPFC voltage initial conditions				Iterations
	Magnitude (p.u.) (All phases)	Phase angle (degress)			
		A	B	C	
1	0.001	0	240	120	11
2	0.25	0	240	120	10
3	0.5	0	240	120	NC
5	0.0502	-87.74	-207.74	32.36	8

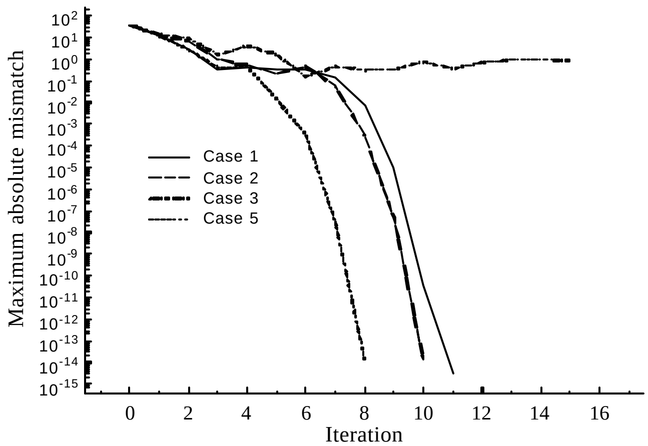

Fig 8. Mismatches as function of number of iterations.

From these results it is clear that an improper selection of initial conditions degrades Newton's quadratic convergence, or more seriously, cause the solution to oscillate or even diverge. It has been found that in some cases the UPFC initial conditions computed by (10) and (11) increase the number of iterations to converge. However, they increase the probability of solving cases that could be divergent when the UPFC initial conditions are chosen based on a heuristic criterion .

Figures 9 to 11 shown the UPFC voltage magnitude profile as a function of the iteration.

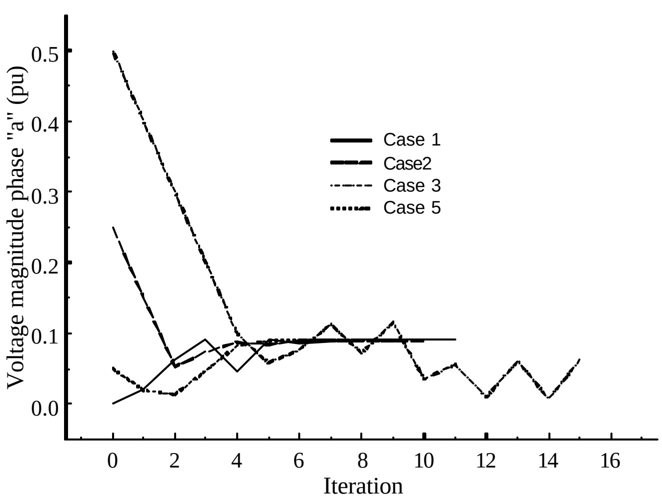

Fig. 9. UPFC Voltage magnitude at phase "a".

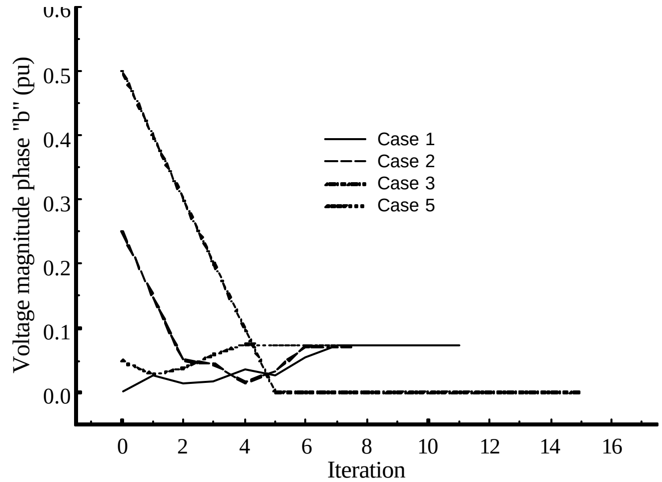

Fig. 10. UPFC Voltage magnitude at phase "b".

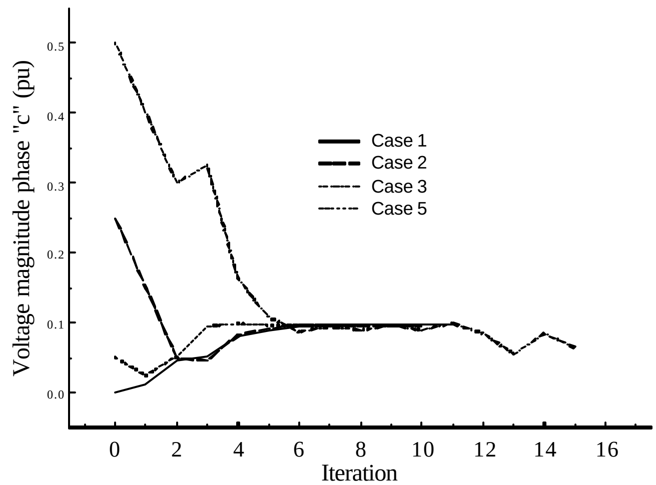

Fig. 11. UPFC Voltage magnitude at phase "c".

It can be seen from figures 8 to 9 that all convergent cases converged towards the same value following the same patterns at the final stage of the iterative process. In case 3, the improper initial conditions produce that voltage magnitude at phase "b" tends to zero.

6 Conclusions

A new three phase UPFC model appropriate for power flow analysis has been proposed in this paper. The static representation was derived based on its operating theory. A polyphase power flow algorithm suitable to analyse complex power systems with multiple UPFCs has been formulated and implemented in a digital computer. In this algorithm, the UPFC control parameters are combined with the nodal voltage magnitude and angles in a single frame-of-reference for unified iterative solutions via Newton's method. The UPFC state variables are adjusted automatically so as to satisfy specified power flows.

The computational efficiency of the power flow has been show by numeric examples. The results clearly show the algorithm's flexibility and reliability towards quadratic convergence. A balanced benchmark network has been used to verify and confirm the correctness of the UPFC model and its implementation. Both, balanced and unbalanced networks have been used to assess the UPFC control capabilities.

Guidelines for the practical UPFC implementation and its adjustment within the NR algorithm have been described. Closed-form equations have been proposed to obtain suitable UPFC initial parameters in order to overcome convergence problems. The influence of both the state variable adjustments and UPFC initial conditions on the convergence process was investigated.

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